Área 1 – Teoria Econômica e Aplicada

PUBLIC SECTOR OVERALL EFFICIENCY: AN APPLICATION IN THE CEARÁ STATE - BRAZIL.

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ABSTRACT

This paper presents two novelties. Firstly, we create an index to represent the overall efficiency of the public sector. We estimate the public sector efficiency levels of the Ceará state by taking into account two ways of efficiency, which are related to expenditure and tax collection. Secondly, we adjust a regional CGE model to accommodate these two types of efficiency. The CGE model was calibrated to represent the economy of the Ceará state. Based on the efficiency estimated, we perform simulation exercises based on two strands, changes in each way of efficiency considered. Regarding to the efficiency results, the sectors of education and health, as well as the tax collection side, show a net efficiency gain in the range studied (2005-2015). The most outstanding result among them is in tax collection, representing an 18.5% net gain. The results from the CGE simulation corroborate to the expectation. Increasing efficiency in the public sector improves the well-being of the households. We emphasize that the results are different for different profiles of households. We hope this paper can provide an alternative point of view to the policymakers and contribute to the public debate concerning public sector efficiency issues.

Keywords: Efficiency. DEA. Public Sector. Regional CGE model.
JEL Codes: C68, H75, R50

RESUMO

Este trabalho apresenta duas novidades. Primeiramente, criou-se um índice para representar a eficiência total do setor público. Estimou-se os níveis de eficiência para o Estado do Ceará considerando duas formas de eficiência, relacionadas às despesas e à receita. Segundo, um modelo de CGE regional foi adaptado para acomodar esses tipos de eficiência supracitados. Baseando-se nas eficiências estimadas, exercícios de simulação foram realizados modificando os níveis de eficiência. Em relação aos resultados acerca da eficiência estimada, os setores de Educação e Saúde tão bem como a coleta de impostos apresentaram ganho líquido de eficiência no período analisado, de 2005 a 2015. Destaca-se o ganho líquido de 18,5% na eficiência em coleta de impostos. Os resultados das simulações do modelo CGE seguem o esperado. Aumentos de eficiência no setor público gera melhoria no bem-estar das famílias. Frisa-se que resultados diferentes são obtidos para os diferentes perfis de famílias adotado. Espera-se que este trabalho forneça alternativas para os tomadores de decisão e contribua para o debate público acerca desse tema.

Palavras-Chave: Eficiência. DEA. Setor Público. Modelo CGE Regional.
Classificação JEL: C68, H75, R50**.**

1 INTRODUCTION

Brazil has one of the highest tax burdens in the world. Official data shows that it was around 32.62% and 33.58% of the GDP in 2017 and 2018, respectively. This amount is similar the average of OECD's members. Despite not only the high tax burden but also the enlargement of it over the last few years, Brazil has not been achieving satisfactory outcome performance. Additionally, it has been facing successive fiscal deficits since 2014. This contributes to a continuous enlargement of the public debt. Given the way the Brazilian central government manages and distributes the revenues, the described scenario directly interferes with the lower tiers of government, such as state-level governments.

Based on data from the World Bank, in 2017 Brazil presented a homicide rate[1] of 30.5. It makes Brazil one of the countries with the highest homicide rate in the world. Considering the context of the Brazilian states, in 2016 the homicide rate in the Ceará state was 40.63, which represents the tenth highest homicide rate. Observing a range from 1998 to 2016 it is easy to realize a slightly positive trend on it within the range from 1998 (13.42) to 2011 (32.30). However, it faces a sharp increase in 2012 achieving a peak of 52.31 in 2014, then it changes the trend and starts to fall achieving the abovementioned value of 40.63.

In terms of education, according to data from the OECD's Programme for International Student Assessment – PISA 2015 report[2], Brazil is between the last ten positions in the three assessed school fields, which are Science, Reading, and Mathematics with the respective scores of 401, 407, and 377. The average and maximum values obtained are respectively 493 and 556 in Science, 493 and 535 in Reading, and 490 and 564 in Mathematics. Going down to state-level analysis, the Ceará state shows a relatively higher score among Brazilian states in an index created by the central government to assess the quality of education, called the Basic Education Development Index IDEB.

In nutshell, the Brazilian government collects as a developed country and delivers goods and services as a poor one, which directly implies efficiency analysis. An International Monetary Fund report, IMF (2015), estimates inefficiency levels of investment on public capital on countries split by income substrates. It shows that the poorer the country, the higher the inefficiency level. This report classifies Brazil as a medium-income country or a developing country. This group presents a 27% efficiency average. Once there are different tiers of government in Brazil, the state-level governments are responsible for part of these results.

Pritchett (2000) emphasizes the importance to insert efficiency issues into the public capital analyses. Otherwise, the analysis might be overestimated. He states it is not correct at all believe that each monetary unit spent by the government will be converted in the same amount of public capital which can be evaluated. Furthermore, the public sector efficiency is not a major problem in countries where the government holds either a smaller investment share or a higher efficiency level. Otherwise, these issues should be taken into account. According to Berg et al. (2015), there are some ways the public sector can demonstrate different efficiency levels such as corruption, resource misallocation, and wasting from failure in planning and execution of public projects and programs.

Even though the literature of assessment of outcomes and/or efficiency of the public sector by looking at the expenditure side is well developed, the assessment of efficiency performance in tax issues has not been so developed so far. Nevertheless, there

[1] The total of deaths caused by violent acts divided by 100.000 inhabitants.
[2] Available at http://www.oecd.org/pisa/.

is a lack of research by considering simultaneously the efficiency of these two sides of the public sector.

This paper, therefore, aims to fill this gap by contributing to the literature with two novelties. First, due to our intention to express a holistic government efficiency level, it creates an index by compiling the efficiency levels of the two government core duties, which are the expenditures and the tax collection. Second, it creates a regional CGE model taking into account these two sides of efficiency. After that, we apply these two novelties in the Ceará state. The former estimates its efficiency performances and the latter uses them as efficiency parameters into the CGE model to see well-being effects.

In regional CGE analyses, a broad amount of models uses a single region approach to make their empirical studies. Even though single region models appear simple, they are useful tools that can provide many answers for specific research questions and specific target units. A well-known single region model is the AMOS model, presented by Harrigan et al. (1991). A recent adaptation of the AMOS for the Illinois state is the AMOIL, presented in Turner et al. (2012).

Due to the omission of interregional feedbacks, Lofgren and Robinson (2002) are skeptical about the accuracy of the results provided by single region models. However, empirical papers, such as McGregor, Swales, and Yin (1999) and Seung et al. (2000), have found that the spillover effects are not significant for regions representing less than 10% of the national economy. Such regions have no economic power to change other economies. Considering that Ceará's economy only represents 2% of the Brazilian economy (IPECE (2016)), the application of a single region model, in this case, is appropriate.

Governments take part as an important stakeholder in the economy, especially in poor regions concerning to promote interventions to reduce social problems such as poverty and inequality. However, there are no general rules to identify properly their roles. One way the government might take action is by increasing its efficiency levels in both sides, expenditures and tax collection. The former allows the government to provide more public goods and services by spending the same amount of resources or even less, and the latter can permit them to obtain more resources.

Beyond this introduction, this paper unfolds as follows. In the next section, we treat the efficiency issues by describing the method and the data used and by estimating the efficiency levels. In addition, in order to provide more information, comments about these estimations are made as partial results. In section three, we derive and describe the regional CGE model. In section four, we perform some empirical exercises by analyzing the Ceará state and interpret the results. After that, we highlight the main topics and the most relevant results as final remarks. Moreover, to conclude it, some further extensions are drawn.

2 PUBLIC SECTOR EFFICIENCY IN THE CEARÁ STATE

2.1 Data and Method of Estimation

The Data Envelopment Analysis, henceforth DEA, is a method to estimate performance efficiency that has been widely used for performance assessment. The main models to measure efficiency by using DEA come from Charnes et al. (1978) and Banker et al. (1984). Under different assumptions, the former using constant and the latter using variable returns-to-scale, both are standards in the performance measurement literature. Although these models are considered the most renowned, Farrell (1957) is also known as a seminal contribution.

Since Charnes et al. (1978), a plethora of studies have been developed concerning this method. Some of them are theoretical breakthroughs, however the biggest part are

applications. Furthermore, these mainly assess the efficiency and productivity of both public and private sector activities. Emrouznejad and Yang (2018) analyze the evolution of DEA-related studies by covering a time range from 1978 to 2016. They focus on criteria such as statistics about the growth of publications and the most widely used academic journals.

As pointed out in the introduction, the main topic of this paper is the public sector efficiency, specifically in the state of Ceará. Additionally, one of the objectives is to estimate its efficiency levels, which will be used as parameters into the CGE model. We estimate these efficiencies by using the DEA methodology, and official government data. Despite some papers pointing out fragilities with the DEA method, Liu et al. (2016) insists that papers applying it have recently grown. Estimating efficiency based on DEA may be considerate a suitable approach here due to the fact the comparison is made with similar institutions, which are other same tier governments[3]. This approach of comparison achieves a more accurate result in terms of potential efficiency than other papers that compare efficiency levels between public and private sectors.

Due to our intention to express a holistic government efficiency level, we applied an approach that takes into consideration the two core duties of the government, which are the expenditures and the tax collection. Besides, we analyze how it behaves over the years. This approach requires a certain range of all data. Therefore, the range from 2005 to 2015 was chosen. However, the cross-sections are each two years, making it a six years sample[4]. The main reason is that one important output is measured every two years[5] and the data needs to be merged hence this feature demands an intersection among them.

For the expenditure side, multiple inputs and outputs from three government sectors are used, which are Education, Health and Sanitation, and security. We select these sectors based on their importance in the current public debate and agenda as well as the fact they represent an average share of around 72% of the state-level government total expenditures. For the tax collection side, on the other hand, we use only one input, the GDP, and one output, the total amount of collected tax, which includes all the jurisdictional taxes the state government tier is accountable for collecting. In order to make the inputs and outputs comparable among the states, we transform them in terms of variables *per capita*.

Since the expenditure side of our analysis is much more complex than the tax collection side, we describe them first. We consider one input by sector, which is the total expenditure in each one of these sectors, in *per capita* terms. Regarding to the outputs, for the security sector we use the homicide rate[6]. For the health and sanitation sector, we use multiple outputs directly related to both health and sanitation. The health-related outputs[7] are Child Mortality, Fetal Death rate[8], Hospital beds, and ambulatories[9]. The sanitation-related outputs, in turn, are the coverage of both sewer and drinking water. For education, we also use different outputs taking into account both quantitative and qualitative measurements. In terms of quality of education, we use the Basic Education Development Index – IDEB[10]. Regarding the quantitative measurement, in turn, we use the attendance rate and the Age-grade distortion. The collected data referred to both

[3] It uses the Brazilian states as the components to generate the frontier.
[4] $t = \{2005, 2007, 2009, 2011, 2013, 2015\} = \{1, 2, 3, 4, 5, 6\}$
5 The Basic Education Development Index – IDEB.
[6] Collected in the official government database IPEADATA.
Available in http://www.ipeadata.gov.br/Default.aspx
[7] These inputs use data collected from the official site of the Central Government's Ministry of Health in its official database called DATASUS. Available in http://www2.datasus.gov.br/DATASUS/
[8] The number of unborn deaths divided by the total of pregnant women.
[9] Both numbers, hospital beds and ambulatories, were population-normalized.
[10] Collected in an official government site linked with the Ministry of Education of the Central Government. Available in http://ideb.inep.gov.br/

quantitative and qualitative measurements consider only the high school level. This is due to the state-level government is its main sponsor.

The public expenditures used as inputs are applied in two different sorts of spends, public sector workers salaries, which is classified as a flow component, and investment and maintenance, which can increase the stock of public capital. Based on the latter component, it is highly likely that the outputs are influenced not only by the inputs from the current year but also by those spent along the previous years. Thus, a simple average of spends from the previous years within the range is used as the input in each sector, $s = \{Education, Health, Security\}$. Therefore, the input of the sector s, in state i, at time t is synthesized as following.

$$INPUT_{s,i,t} = \frac{1}{t}\sum_{j=1}^{t} Input_{s,i,t}\,, for\ t \leq T, and\ T = 1, \dots, 6.$$

Although the fact the chosen inputs have the same measure, which is the total amount spent in that sector, the outputs present different sorts of scales making them incompatible. In order to overcome this issue, they are standardized by using the so-called Min-Max Scaling method.

In terms of expenditures, efficiency levels are estimated both ways by sector individually and by combining them into a single measure. The latter bases on an assumption in which those three sectors used are able to represent the efficiency in expenditure as a whole. In turn, this assumption is based on that together they account for around 72% of the total expenditures of the target region. Combining the ideas of using multiple inputs and outputs with estimating a single expenditure-side efficiency parameter requires a way of weight both of them.

This procedure is made as a two-steps weighting method. In the first step, we create a single output for each sector by applying a simple average on its respective multiple outputs, which means giving them the same importance, hence equally weighting the outputs. In the second step, we create a single input and a single output for the public sector. The former comes from the sum of the multiple inputs, and the latter, in turn, is acquired by weighing them based on the shares of the total resources implemented in each specific sector. In this case, it is assumed that these shares might be considerate as the government concerns in each sector. In other words, the more the government spends in a sector, the most important it assumes it is. Both the first step and the input side of the second step are easily understandable. However, the following expression might be necessary to make the output side of the second step clear enough.

$$OUTPUT_{i,t} = \sum_{j} \omega_s\, Output_{i,t,j}\,, \qquad j = Education, Health, Security.$$

Where i denotes the state, s denotes the sectors, and ω the aforementioned weighting shares.

2.2 Efficiency Measure and the case of Ceará State

Firstly, both sides of the public sector efficiency are estimated to the state of Ceará using the already specified range. However, the expenditure side estimation is split in two ways. First by sector and then those formulas mentioned in the previous section are used to provide a compiled efficiency level of the expenditures.

Furthermore, since this paper aims to combine the efficiencies into a compiled measure, we calculate a simple average and bring forth a total efficiency level, which

takes into account both sides of efficiency each year. Given its features, we call it the Two-way Efficiency Index, henceforth TWE, as shown below:

$$TWE_t = 0.5 * (\epsilon_{col,t} + \epsilon_{exp,t})$$

At time t, $\epsilon_{col,t}$ and $\epsilon_{exp,t}$ are respectively the efficiency levels in tax collection and expenditures. All the results are shown in table 1.

Table 1: Efficiency Levels in Expenditure and Tax Collection, and the TWE index.

Year	Education	Health	Security	Expenditure	Tax	TWE
2005	0.592	0.551	0.894	0.679	0.748	0.714
2007	0.607	0.622	0.897	0.708	0.679	0.694
2009	0.664	0.580	0.759	0.668	0.738	0.703
2011	0.711	0.595	0.616	0.641	0.812	0.727
2013	0.637	0.599	0.206	0.481	0.899	0.690
2015	0.684	0.557	0.169	0.470	0.886	0.678

Source: the authors.

At a glance, the negative trend in the evolution of efficiency in the security sector demonstrates an erratic behavior due to a sudden decrease in the last two years. Although in the first two years it seems constant, there is a clear reduction in the middle of the range and then a sharp decrease during the final years. As mentioned in the introduction, a noteworthy increase in the homicide rate had occurred since 2012 in our target region. Nevertheless, given there are other exogenous factors influencing efficiency issues, it is worth mentioning that it is highly unlikely the government can react as fast as this sudden change requires by adjusting its efficiency.

Despite the fluctuations and considering the whole range, the sectors of education and health, as well as the tax collection side, show a net efficiency gain. The Health sector presents the slightest net gain and the tax side the biggest one. The latter represents an 18.5% gain. Regarding the TWE, although there is no observed pattern while the first two-thirds of the range, there is a loss in the total efficiency in the last two years. The sharp reduction in the security sector is the main responsible for this result.

The results regarding the efficiency levels are useful by themselves. They also fit as an assessment tool by illustrating in which sector the government needs to ameliorate its activities. Besides, different ranges may be used for specific purposes. A four years range can be applied for analysis focused on government mandates for instance. The results can be used as a prior step to assist other analysis as well. A suitable way to understand what are the possible implications of improvements in the public sector efficiency in the economy as a whole is by simulating them in a CGE model. The next section shows a CGE model adapted to suit these specifications.

As previously mentioned, efficiency depends on public sector efforts as well as some exogenous factors. Considering the former one, a part of the total efficiency can be enhanced by improving the government functioning. A simulation of that is performed in section 4.

3 A CGE MODEL FOR THE CEARÁ STATE

The following specified model is adapted from Hosoe et al. (2010). We made three major changes. Firstly, we split the households in two income-based profiles. In order to include the consumption of public goods as part of the total consumption of households, we assume the government as also a provider of goods and services. It aims to capture

public policy effects in different substrates of the population. Secondly, we split the foreign sector in two different regions. Even though this change seems unnecessary for the purpose here, it must give us a more accurate and realistic way to deal with trade issues in a state-level unit as the target region, which is subject to exchange rate related to the foreign trade but not to the internal trade. Additionally, this characteristic might be used for further studies or even as a reference model. Finally, we insert efficiency parameters in both the consumption of public goods and tax revenue. One of the key points of this paper, the latter improvement is a contribution to the CGE literature. The model description splits into the following subsections.

3.1 Households

The first type (r) acquires income from the factors' rentals. After paying the income tax (T_H^D), the households choose the amounts of private consumption (C_i) of each one of the supplied goods, and savings (S^S). The second type (p) belongs to a social security program and only receives income from government transfers (tr_H). In addition, the government provide an amount of public services to them (G_i).

Based on a CES[11] function, the households' utility function comes from the optimization process, taking into account the constraint on available income, as shown below.

$$\max_{C_{r,i}} UU_r = \left(\sum_i \alpha r_i C_{r,i}^{\rho} + \epsilon_{exp} \sum_i \mu G_i^{\rho}\right)^{\frac{1}{\rho}} \quad s.t. \quad \sum_i p_i^{qF} C_i^R \leq \sum_h p_h^f FF_h - S^S - T_H^D \tag{1}$$

$$\max_{C_{p,i}} UU_p = \left(\sum_i \alpha p_i C_{p,i}^{\rho} + \epsilon_{exp} \sum_i \mu G_i^{\rho}\right)^{\frac{1}{\rho}} \quad s.t. \quad \sum_i p_i^{qF} C_i^P \leq tr_H \tag{2}$$

Where p_h^f and p_i^{qF} are the prices of factors and sectorial goods respectively, FF_h is the endowment of factors. αr_i and αp_i are the share parameters of the households in the utility function, r and p respectively. In turn, ϵ_{exp} is the efficiency parameter, μ is a weight for the consumption of public goods[12], and ρ is the elasticity parameter.

3.2 Sectors

Each sector works as a profit-maximizing firm. As a result, it uses an optimal level of both factors (F_h) and intermediate inputs $(X_{j,i})$, which are combined with the composite factor (Y_i). Due to this structure, the production splits into two steps:

First step:

$$\max_{F_{h,i}} \pi_j^y = p_i^y Y_i - \sum_h p_h^f F_{h,i} \ , \qquad \text{s.t.} \quad Y_i = b_i \prod_h F_{h,i}^{\beta_{h,i}} \tag{3}$$

Second step:

$$\max_{Y_i, X_{j,i}} \pi_i^z = p_i^z Z_i^S - \left(p_i^y Y_i + \sum_j p_j^{qF} X_{j,i}\right), \qquad \text{s.t.} \quad Z_i^S = \min\left(\frac{X_{j,i}}{ax_{j,i}}, \frac{Y_i}{ay_i}\right) \tag{4}$$

p_i^z, p_i^y, and p_j^{qF} are the prices of domestic goods, the composite factor, and the composite good, respectively. b_i is a scaling coefficient, and $\beta_{h,i}$ is a share coefficient in

[11] Constant Elasticity Substitution – CES.

[12] It is assumed that households attribute a higher value to the private consumption than to the consumption of public goods. It is assumed $\mu = 0.5$.

the production function. Also, $ax_{j,i}$ and ay_i are input requirement coefficients for one unit output.

3.3 Armington's assumption and Foreign Trade

Combining some model attributes such as be an open and a small economy suggests that the analyzed region has no significant impact on the other regions. Therefore, both the export and import prices are exogenously given.

We use two different sets of price variables. One considering the domestic currency (p_i^{xW} and p_i^{mW}) and the other the foreign currency ($p_i^{U\$xw}$ and $p_i^{U\$mw}$). These currencies are related to each other by a marketability margin (mg^w), which also includes the exchange rate.

According to the Armington's assumption, the final consumers, including the sectors, acquire not sectorial goods but rather the Armington's composite good (Q_i^F), which represents a *mix* composed by national (M_i^C) and foreign imports (M_i^W) and local sectorial goods (Q_i^S). The optimization problem for the virtual sector of the i-th Armington's composite good is:

$$\max_{M_i^C, M_i^W, Q_i^S} \pi_i^{qF} = p_i^{qF} Q_i^F - \left[p_i^{qS} Q_i^S + p_i^{mC} M_i^C + (1+\tau^m) p_i^{mW} M_i^W\right] \quad (5)$$

$$s.t. \quad Q_i^F = \gamma_i \left(\delta q_i^S \left(Q_i^S\right)^{\eta_i} + \delta m_i^C \left(M_i^C\right)^{\eta_i} + \delta m_i^W \left(M_i^W\right)^{\eta_i}\right)^{\frac{1}{\eta_i}}$$

p_i^{qS}, p_i^{mC}, and p_i^{mW} are the supply prices of locally offered goods, imports from the rest of Brazil, and from the rest of the world, respectively. τ^m is an import tax, γ_i is a scaling coefficient, and η_i is a parameter defined by the elasticity of substitution. Furthermore, δq_i^S, δm_i^C and δm_i^W are input share coefficients.

On the other hand, it is also necessary to analyze the strategic decisions by taking into account the supply. The decisions are related to what is internally supplied (Q_i^S) to the rest of Brazil (X_i^C) and to the rest of the world (X_i^W). EVEN We assumed that the sectors divide the domestic output by using an imperfect transformation process, based on a CET function. Every sector adjusts their output for both domestic and foreign consumption.

$$\max_{Z_i^S, X_i^C, X_i^W, Q_i^S} \pi_i^{zS} = \left(p_i^{qS} Q_i^S + p_i^{xC} X_i^C + p_i^{xW} X_i^W\right) - (1+\tau_i) p_i^z Z_i^S \quad (6)$$

$$s.t. \quad Z_i^S = \theta_i \left(\xi q_i^s \left(Q_i^S\right)^{\phi_i} + \xi x_i^c \left(X_i^C\right)^{\phi_i} + \xi x_i^w \left(X_i^W\right)^{\phi_i}\right)^{\frac{1}{\phi_i}}$$

p_i^{xC} and p_i^{xW} are the export prices for the rest of Brazil and for the rest of the world, respectively. τ_i is a production tax on the locally produced good (ICMS plus Others). θ_i is a scaling coefficient, and $\phi_i$is a parameter defined by the elasticity of transformation. Additionally, ξq_i^s, ξx_i^C, and ξx_i^w are share coefficients.

3.4 Government

The government is assumed to collect taxes to fund its aforementioned expenditures. Due to its second way to express efficiency, there is a parameter of efficiency in each one of the tax collection channels. Thus, it collects a direct tax from the r households' income (T_H^D), from production, based on the ICMS tax (T_i^{ICMS}) and the

Other tax (T_i^{OT}). In addition, it collects the imports tax (T_i^M). The tax rates are expressed as τ_H^D, τ_i^{ICMS}, τ_i^{OT}, and τ_i^m, respectively. Additionally, the efficiency parameters associated with them are, respectively, $\epsilon_{col,d}$, $\epsilon_{col,icms}$, $\epsilon_{col,ot}$, and $\epsilon_{col,m}$. Equation 11 shows the total revenue collected. This amount is already carrying within its composition the efficiency levels from all of the sources of collecting. In this case, it can be generalized as the collecting efficiency level, ϵ_{col}.

The government spends its tax-revenues in transfers to the p households (tr^H), on consumption (G_i^F), and savings (S^G). It also consumes each produced good in a constant ratio (μ_i). The key equations for the government are:

$$T_H^D = \epsilon_{col,d}\, \tau_H^D \sum_h p_h^f FF_h \tag{7}$$

$$T_i^M = \epsilon_{col,m}\, \tau_i^m p_i^{mW} M_i^W \quad \forall i \tag{8}$$

$$T_i^{ICMS} = \epsilon_{col,icms}\, \tau_i^{ICMS} p_i^z Z_i^S \quad \forall i \tag{9}$$

$$T_i^{OT} = \epsilon_{col,ot}\, \tau_i^{OT} p_i^z Z_i^S \quad \forall i \tag{10}$$

$$T_{Total} = T_H^D + \sum_i (T_i^M + T_i^{ICMS} + T_i^{OT}) \tag{11}$$

$$G_i^F \leq \frac{\mu_i}{p_i^{qF}} \left(T_H^D + \sum_i (T_i^M + T_i^{ICMS} + T_i^{OT}) - S^G - tr^H \right) \forall i \tag{12}$$

3.5 Investment and Savings

Even though we have no adequate way to consider the investment variable in this model due to its static modeling limitations, it is necessary to insert it in order to guarantee the macro closure conditions. Thus, we assume there exist an investment agent who collects resources from the r households, the government, and the foreign sector. This agent spends them by purchasing investment goods in a proportionally constant share.

3.6 Macroeconomic Closure Conditions

Once the model describes different sorts of behavior, we have to impose the Market conditions in order to achieve an equality between supply and demand in every market. This macroeconomic closure characterizes the model as a neoclassical model. By using Walras' Law, we can choose a *numeraire*[13]. Since the endowment of the productive factors are fixed, we follow standard model and use the price of the labor factor.

3.7 Calibration

The model requires values for the parameters and exogenous variables. These values are calibrate by using a Social Accounting Matrix – SAM based on data from an Input-output table of the Ceará state taking into account the 2013 base year. The calibration process also requires some extra information, obtained from a specific

[13] One price in which all of the other prices are comparable.

government report, which is IPECE (2016). In terms of production, the SAM is split in 6 productive sectors[14]. Table 2 shows the sectors.

Table 2: Codes for short denomination of the sectors.

Code	Sectors
S1	Agriculture, including support for agriculture and post-harvest.
S2	Extractive industry.
S3	Transformation Industry, Building, Power Electricity, Water and Sewage, and Others.
S4	Trade and repair of motor vehicles and motorcycles, transportation, storage and mail, accommodation and food.
S5	Private Services.
S6	Management, security, public education and health, and social security.

Source: the authors.

Concerning the key parameters of efficiency, to make the calibration procedure compatible at all, we use the efficiency levels from 2013 as the efficiency parameters. As shown in section 2, the efficiency levels estimated to the Ceará state are 0.481 in expenditures and 0.899 in tax collection. Therefore, the calibrated parameters link the theoretic model to the economy of Ceará state.

4 EMPIRICAL SIMULATION EXERCISES

This paper focuses on public sector efficiency issues, precisely efficiency in two sides of the government duties, which are the expenditures and tax collection. Thus, it is suitable performing simulation exercises by changing the efficiency levels in both sides. The Ceará state is the target unit as well as it was in section 2. In order to achieve more accuracy with analysis, exercises are performed individually focusing on one side at a time.

As the analysis of the expenditure side is more straightforward, it is the first to be taken. Given the sharp reduction in the expenditure efficiency level from 2011 to 2013, we suppose the government found a way to enhance its efficiency to the level it had presented in the previous estimated period, 2011. Thus, the simulation consists of changing the efficiency level from the base year value to 0.641, which represents a positive percentage variation of 33.2%. It means that the government becomes a more effective provider of public goods and services.

It is worth to mention that, by construction, this kind of intervention does not change significantly variables other than the consumption of public goods. Thus, the results of the macroeconomic variables can be overlooked without losing any analytical accuracy. Therefore, we focus on the variations in well-being[15].

Since different Households are taken into account, we have to pay attention to their well-being variation separately. Reminding, increasing efficiency means providing more public goods and services with no additional costs. Thus, the expected result is an increase in the utility function of both of households. As a result, a positive equivalent variation is observed. The results from this simulation are shown in the middle row in table 3.

[14] The disaggregation level depends on the target analysis. In this case, a low level of disaggregation does not affect the analysis.

[15] The Hicks Equivalent Variation is used as a measure of well-being. It is based on both private consumption and consumption of public goods and services.

Table 3: Equivalent Variation from changes in the expenditure efficiency levels.

Efficiency Parameter	Δ%*	p Household	r Household
0.470	-2.28	-6.23	-3.51
0.641	33.2	64.43	38.93
0.899	86.9	134.64	85.12

Source: the authors.
Note: *Percentage variation from 2013 base year.

Assuming the government has a big bold goal to increase its expenditure efficiency to the same level as it has by collecting taxes. Thus, we simulate a change by increasing the efficiency level to 0.899, which represents an 86.9% improvement. Perhaps, implementing a variation of almost 87% seems unfeasible. However, one can understand that as a sort of sensibility test. As expected, the results show an increase in the well-being of the households larger than those achieved by the smaller change previously shown. Additionally, we highlight that the p households obtained a gain larger than r huseholds do.

Once there was an observed reduction on the efficiency level we are working on from 2013 to 2015 and also due to the need for a sensibility test in a different direction, a negative change must be considerate as a third scenario. Thus, we simulate a reduction to the level estimated for 2015, which is 0.47 and represents a 2.28% reduction. Again, the results confirm the expect direction of change in the well-being. Both types of households lose part of the consumption of public goods what directly affects their well-being. The p households, in this case, face a bigger loss comparatively the other type of households.

A caution that is worth mentioning is the observed pattern concerning the magnitude of variation. The p households always obtain a higher impact than the r households, regardless of the direction whether positive or negative. In other words, positive changes allow them to enjoy the highest gain whereas negative changes bring forth the highest loss. It occurs due to the fact the consumption of public goods represents a higher share of their total consumption relative to the r households. Reminding, in this paper the p profile is a set of low-income households hence they tend to consume more public goods than the richest households. It shows the model is appropriately calibrated.

Now, the analysis keeps constant the efficiency in expenditure and focus attention on the tax collection side. Unlike the previous analysis, this feature shakes the economy more deeply. It means there are more channels to spread the impacts of a shock making them reverberate both directly and indirectly on a set of different variables. For instance, a change in efficiency direct implies a change in revenues, which in turn changes the amount of public goods and services provided. However, we will keep the focus on well-being analysis.

For simplicity, it is assumed that the efficiency levels in all of the collection venues have the same level. This assumption is required because the estimation in tax collection was aggregately made, reaching a magnitude of 0.86 in the base year. Following the exercises made in the previous analysis, different directions of variations are performed, which works simultaneously as sensibility analysis.

The exercises are split in shocks in two sources of collection, income and production, which can be seen as the way they were modeled, direct and indirect, respectively. Since the base year shows the highest level of the chosen range, only two shocks are performed, one negative and one positive. Table 4 shows both shocks, the negative are presented in the first rows and then the positive ones. Reminding that $\epsilon_{col,d}$ is the tax collecting efficiency over income, which is made directly. In the case of the indirect tax collected from production, we assume the efficiency equally changes in all channels. Thus, we use ϵ_{col}, where $\epsilon_{col} = \epsilon_{col,i} = \epsilon_{col,o} = \epsilon_{col,m}$. Results are shown on table 4.

Table 4: Equivalent Variation from changes in the tax-collection efficiency levels.

Efficiency Parameter	Δ%*	p Household	r Household
$\epsilon_{col,d} = 0.812$	-10	-0.004	0.007
$\epsilon_{col} = 0.812$	-10	-2.07	-3.59
$\epsilon_{col,d} = 0.943$	5	14.08	22.61
$\epsilon_{col} = 0.943$	5	14.38	21.29

Source: the authors.
Note: *Percentage variation from 2013 base year.

For the negative shock, we suppose an event in which the efficiency falls to the level that Ceará had presented in 2011, which was 0.812. This represents a reduction of around 10% relative to the base year. Results show that a loss of efficiency in the collection of the direct tax implies an insignificant variation of the well-being of both agents. Although the effects in well-being are tiny, we underline that they are divergent, which means the p households reach an expected negative variation, but the r households get an unexpected positive change. Loss is the expected effect due to the expected reduction in tax revenue and, as a result, a reduction in the provision of public goods, which are part of their consumption. Analyzing the negative variation in the indirect collection, conversely, the results present the expected negative effects for both households.

Two reasons are likely to be contributing to this divergence. Firstly, the small share of the revenues coming from this category of tax collection makes a slight impact on the provision of public goods. This amount of resources is proportionally small compared to the collection from production. Secondly, the consumption of public goods is not the only source of benefit these households may achieve from this variation. Additionally, it is less important than private consumption. It does not mean there was no negative effect. It just mean this likely negative effect was not enough to overcome the positive one. As a result, the net gain is positive in the first case. Similarly, the results from the shock in indirect tax might be explained. In this case, however, the impact is large enough not only to overcome any positive effects but also to be even worse than those p households face.

On the other hand, the positive shocks are performed by assuming a magnitude of half of those assumed about the negative case. This is due to the current high level of efficiency in tax collection, which is almost 90%. Adding a 10% efficiency gain the level would achieve nearly 100%, which is hard to reach. In this case, the results show an expected gain of well-being regardless the types of households. Since the provision of public goods increases with the enlargement of the tax revenue, their consumption increases as well.

In this case, we highlight that the r households obtain the highest gains. This is due to the way they are modeled. One should remind that the r type is more complex and has related variables shaken by the shock other than the consumption of public goods. One example is the private capital that they are endowed and receive income to rent this productive factor. It is some type of knock-on effect.

Even though the exercises performed seems naive, it is suitable for the target analysis. Besides, understanding how the agents respond to these incentives helps the decision-makers to figure out a set of alternatives in which they can base new public poicies. Furthermore, the way the model is adapted contributes to the literature by itself.

5 FINAL REMARKS

We highlight the general contribution of this paper is analyzing the public sector efficiency holistically by taking into account the two-way efficiency in the public sector.

Detailing this contribution, we also highlight the double analysis performed. Firstly, by creating an index to compile efficiency as a whole and secondly by adapting a CGE model to consider this important factor into the public sector issues.

Before going towards the remarks we need to shatter the naive understanding that the unique way to make the government more efficient is by reducing its spending. The efficient government can use tools such as benefit-cost analyses to allocate efficiently resources between their programs and priorities until no net benefits can be had from further re-allocations of resources. Based on this broad understanding we realize that improve efficiency in the public sector is not only feasible but also desirable.

The government takes part as an important stakeholder in the economy, especially when it belongs to a poor region and must promote interventions to reduce common social problems such as poverty and inequality. Reminding, in this paper the p profile is a set of low-income households. Therefore, the government should find ways to assist them. This assistance not necessarily is direct by providing more public services for instance. It can be by achieving more efficiency collecting as well. As a result, the revenues will change implying in many possibilities to spend them.

Sometimes changes in efficiency can achieve similar effects as changes in tax rates. However, unlike increases in the tax rate, efficiency changes can minimize the disturbance created by changing the incentives of the productive sector. Additionally, performing policies to increase efficiency probably works as a way to signal credibility and reach trustworthy, desired features for an agent able to promote economic incentives.

Another noteworthy issue is that changes in efficiency is feasible of implementation. It can be even easier than apply tax reductions for example. Tax policies always require a huge political effort, mostly in the current days in which the Brazilian central government is facing problems to balance the budget. Therefore, finding a way to work efficiently creates positive effects. These effects can be used to guide political decisions about these issues. Keeping the same tax levels, the extra revenue coming from increasing in efficiency on tax collection can be applied in sectors that creates good incentives for private sector such as infrastructure. The economic benefits of the latter option have been widely shown in the literature[16].

As a direct policy implication of this paper, the policymakers and public managers can work to enhance its efficiency levels in order to provide more of public goods in terms *per capita* and then enrich the well-being of the households. Similarly, one can affirm that becoming more efficient the government can either provide more of public goods in terms *per capita* or provide the same amount to a bigger amount of people, or even both options, it depends on the magnitude of the changes of all factors. Even though this static model is not able to analyze these issues about compensating the population growth, this conclusion is closely related to the former provided. Therefore, it is attainable.

Based on the caveats identified in this paper, some of them due to the static version of the model, we are working in a dynamic version of this model. This version might be able to analyze appropriately other types of simulations. The new results can allow reaching further analysis such as reducing public debt by saving resources from tax collection or using them to increase public investments.

Another extension we are currently working on is estimating efficiency by using another method aiming to achieve more accuracy in those estimations, which is the Linearized Free Disposal Hull – LFDH. Furthermore, we intend to split the households in a different way, one in which both of them are able to work. This new specification put the profile of the households closer to reality and balance the effects of policies.

[16] The most famous reference about it is Aschauer (1989).

REFERENCES

ASCHAUER, D. A. Is public expenditure productive? Journal of Monetary Economics, v. 23, p. 177-200, 1989.

BANKER R.D. and MOREY R.C. Efficiency analysis for exogenously fixed inputs and outputs, Ops. Res., 34, 513-521. 1986.

CAZALS C., FLORENS J.P., SIMAR L. Nonparametric frontier estimation: a robust approach. *Journal of Econometrics*, *106*(98), 1–25. 2002.

CHARNES A., COOPER W.W. and RHODES E. Measuring the efficiency of decision making units, Eur. J. Opl. Res 2, 429-444. 1978.

EMROUZNEJAD, Ali. YANG, Guo-liang. A survey and analysis of the first 40 years of scholarly literature in DEA: 1978 - 2016 . Socio-Economic Planning Sciences 61. P. 4-8 2018.

FARRELL, M. J. The measurement of productive efficiency. Journal of the Royal Statistical Society. Series A (General), Vol. 120, No. 3 pp. 253-290. 1957.

FARRELL, M.J. FIELDHOUSE, M. Estimating efficient production functions under increasing returns to scale, Journal of the Royal Statistical Society. Series A 125, pp. 252-267. 1962.

GILLESPIE, G., J. MCGREGOR, J.K. SWALES AND Y.P. YIN. The displacement and multiplier effects of regional selective assistance: a computable general equilibrium analysis. Regional Studies, v. 35, n. 2, 125-139. 2001

HARRIGAN, F., P.G. MCGREGOR, N. DOURNASHKIN, R. PERMAN, J.K. SWALES AND Y.P. YIN. AMOS: A Macro-Micro Model of Scotland. Economic Modelling, 10, 424-479. 1991.

HOSOE, Nobuhiro. GASAWA, Kenji. HASHIMOTO, Hideo. Textbook of computable general equilibrium modelling: programming and simulations. Palgrave Macmillan. 2010.

IPECE - Instituto de Pesquisa e Estratégia Econômica do Ceará, 2010, Produto Interno Bruto: PIB do Ceará na Ótica da Produção – 2010 – 2013. IPECE Informe. Secretaria de Planejamento e Gestão, Governo do Estado do Ceará. Fortaleza, Ceara, Brasil.

IPECE - Instituto de Pesquisa e Estratégia Econômica do Ceará, 2016, Ceará em Números 2016. Secretaria de Planejamento e Gestão, Governo do Estado do Ceará. Fortaleza, Ceara, Brasil.

LIU, John S. LU, Louis Y.Y. LU, Wen-Min. Research fronts in data envelopment analysis. Omega. 58, 33–45. 2016.

LOFGREN, H. S. ROBINSON, Spatial-network, general-equilibrium model with a stylized application. Reg. Sci. Urb. Econ. 32, 651-671. 2002.

MADDEN, J.R. FEDERAL: A two-region multi-sectoral fiscal model of the Australian economy, PhD Thesis, University of Tasmania, Hobart. 1990.

MCGREGOR, P.G., SWALES, J.K. YIN, Y.P. Spillover and feedback effects in general equilibrium interregional models of the national economy: A requiem for inter-regional input-output? In: Hewings, G.J.D., Sonis, M., Madden, M., (Eds), Understanding and Interpreting Economic Structure. Springer, Berlin, p. 167-190. 1999.

PARTRIDGE, Mark D. RICKMAN, Dan S. Computable general equilibrium (CGE) modelling for regional economic development analysis. Regional Studies, v. 44, n. 10, p. 1311-1328. 2010.

RICKMAN, Dan S. Estimating the impacts of regional business assistance programs: Alternative closures in a computable general equilibrium model. Papers in Regional Science, v. 71, n. 4, p. 421-435. 1992.

SEUNG, C. K., HARRIS, T. R., ENGLIN, J. E., & NOELWAH, R. N. Impacts of water reallocation: A combined computable general equilibrium and recreation demand model approach. The Annals of Regional Science, 34, 473–487. 2000.

THANASSOULIS E., DYSON, R.G. and FOSTER, M.J. Relative efficiency assessments using data envelopment analysis: an application to data on rates